Simulating Binary Neutron Star Mergers

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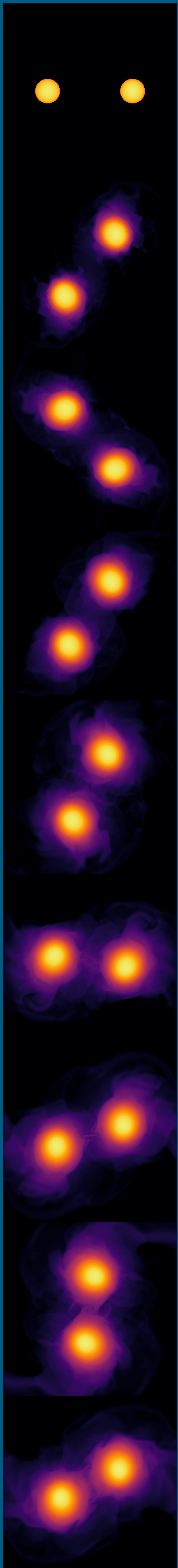
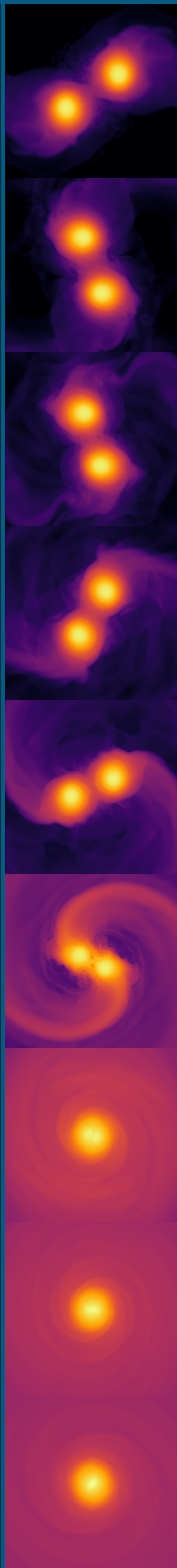




Introduction

Binary neutron star mergers are extremely energetic events. They occur when two neutron stars orbiting each other lose enough orbital energy to crash into each other, combining to create a single object. In order to study these events we need to get an idea of what signals we should be looking for with telescopes and ***gravitational wave*** observatories, and how these signals are affected by the physics taking place which we can do using simulations.

Neutron stars themselves are ***very massive***, very compact objects, squeezing about the same amount of mass as the sun into just a ***20km diameter*** (a teaspoon of neutron star matter would weigh more than the entire human population). They are thought to be composed almost entirely of a fluid of subatomic particles, mostly neutrons, surrounded by a crust of iron.

The extreme energies involved in mergers also mean we expect the matter to be moving at significant fractions of the speed of light. Because of the strength of gravitational fields and speeds involved, our simulations must use general relativity in order to be accurate.

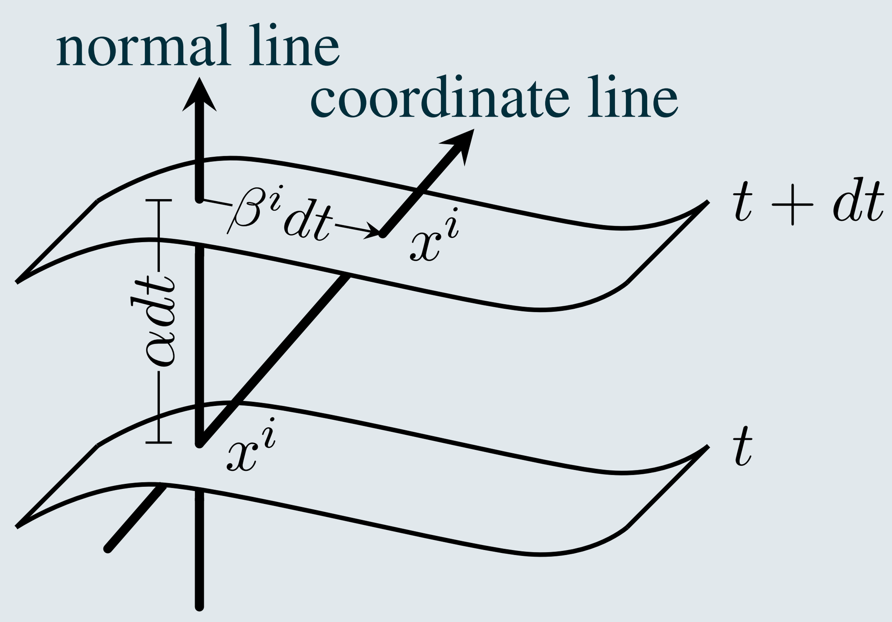
Simulating the Spacetime

Merger simulations are based on the ***Einstein Field Equations***, which are normally expressed through the relation

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu},$$

where $G_{\mu\nu}$ is the ***Einstein Tensor*** which contains information about the curvature of spacetime, and $T_{\mu\nu}$ is the ***Stress Energy Tensor*** which contains information about the matter inhabiting the spacetime. Through this relation, matter tells space how to curve, and space tells matter how to move.

To solve these equations, we must first split the 4-dimensional spacetime into a series of 3-dimensional spatial slices collectively called a ***Foliation*** (see illustration). A foliation is primarily described by the lapse α and shift β functions. The lapse function controls the passage of time between slices, and the shift function can be thought of as the velocity of the coordinate system. By evolving the lapse, shift, and a number of quantites related to the curvature of each slice, a stable evolution of a spacetime can be obtained.



Simulating the Fluid

To model the neutron star matter, we use a ***relativistic perfect fluid***. Such a fluid does not conduct heat, is non-viscous, has isotropic pressure, and obeys the ***relativistic Euler Equations***:

$$\rho h a^\mu = -(g^{\mu\nu} + u^\mu u^\nu)\nabla_\nu p,$$

where a^μ and u^μ are the acceleration and velocity of the fluid, and ρ, h, and p are the density, specific enthalpy (a measure of thermodynamic work) and pressure of the fluid respectively.

The microphysical behaviour of the fluid is encapsulated in the ***Equation of State*** which provides a relationship between the macrophysical variables (e.g. density, temperature, electron fraction, etc.) and the pressure and internal energy. Equations of state can range from simple analytic expressions of pressure in terms of density, to multi-dimensional tables calculated directly by nuclear physics, combined with an interpolation routine.

Extracting a Gravitational Wave Signal

Once the simulation is complete, we can analyse the evolution of the spacetime and extract a gravitational wave signal. For a typical merger event, there are three distinct phases: the ***inspiral*** where the two stars spiral towards each other moving faster and faster, the ***merger*** itself where the two stars collide, and the post merger ***remnant*** where we see the object (either a more massive neutron star or a black hole) that results from the collision.

In the gravitational wave strain plot below we can see these phases: the inspiral is the slow increase of amplitude and frequency up to a peak, the merger is the chaotic period shortly following the peak in amplitude, and the remnant signal (also called a ***ringdown***) is the slow relaxation of the amplitude and frequency as the resulting neutron star settles towards a steady state.